LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – APRIL 2010

ST 2813 - SAMPLING THEORY

Date & Time: 21/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Explain the terms parameter and statistic with a suitable example for each term.
2. Define the two types of Sampling Designs with suitable example for each type.
3. Find $E[I_i(s)]$ and $E[I_i(s)I_j(s)]$; $i, j = 1, 2, \ldots, N$; $i \neq j$ under any sampling design $P(.)$.
4. Derive mean and variance of inclusion indicator under any sampling design.
5. Check whether or not s_y^2 is unbiased for S_y^2 under Simple Random Sampling Design.
6. Explain the Linear Systematic Sampling Scheme and define its probability sampling design.
7. Explain Random Group Method of Sampling and construct an estimator for population total under this method.
8. Show that $\hat{Y}_{LR}$ is more efficient than $\hat{Y}_R$ unless $\beta = R$.
9. Explain the cumulative total method and show that it is a PPS selection method.
10. Describe Multistage Sampling.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. In Simple Random Sampling Design, explain the unit drawing mechanism and prove that the mechanism implements the design.
12. Show that for any fixed size sampling design,

$$(i)\ \sum_{j=1}^{n} \pi_{ij} = (n-1)\pi_i ; \quad j \neq i$$

and

$$(ii)\ \sum_{j=1}^{n} (\pi_i \pi_j - \pi_{ij}) = \pi_i (1 - \pi_i) ; \quad i = 1,2, \ldots, N; \ j \neq i .$$

13. Compare the efficiency of $\hat{Y}_{LSS}$ and $\hat{Y}_{SRS}$ when the population is linear.
14. Why do we use Desraj ordered estimator instead of Horwitz – Thompson estimator under PPSWOR sampling scheme? Is Desraj ordered estimator unbiased for population total? Justify your answer.
15. In Midzuno Sampling Design, prove that the estimated variance of $\hat{Y}_{HT}$ is non-negative for all samples "**s** " receiving positive probabilities.

16. In LSS, when the population is linear, derive Yates's corrected estimator for estimating population total without error.

17. Obtain the estimated variance of $\hat{Y}_{DR}$ for any sample size " n " .

18. Describe Warner's randomized response technique for estimating the population proportion Π_A .

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19 (a) Give an example to show that an estimator can be unbiased under one design but biased under another design. (10)

(b) Obtain the formula for π_i and π_{ij} under SRS Design and hence find $\hat{Y}_{HT}$ and V ($\hat{Y}_{HT}$) . (10)

20 (a) Describe the Regression estimation procedure and find the approximate bias and mean square error of $\hat{Y}_{LR}$. (10)

(b) Obtain the expression for $\hat{Y}_{St}$, V ($\hat{Y}_{St}$) and v ($\hat{Y}_{St}$) under the design (i) SRSWOR (ii) PPSWR . (10)

21. Derive the approximate bias and mean square error of the estimator $\hat{Y}_R$ and hence obtain their expressions under (i) SRSWOR , (ii) PPSWR , and (iii) Midzuno Sampling.

22 (a) A SRS of size $n = n_1 + n_2$ with mean $\hat{\bar{Y}}$ is drawn from a finite population of N units and a SR Subsample of size n_1 is drawn from it with mean $\hat{\bar{Y}}_1$. Find V ($\hat{\bar{Y}}_1$ - $\hat{\bar{Y}}_2$) , where $\hat{\bar{Y}}_2$ is the mean of the remaining n_2 units in the sample. (12) .

(b) Derive the variance of

(i) Hansen – Horwitz estimator in double sampling

(ii) Estimator $\hat{Y}_{TS}$ in Two – Stage Sampling. (8)

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